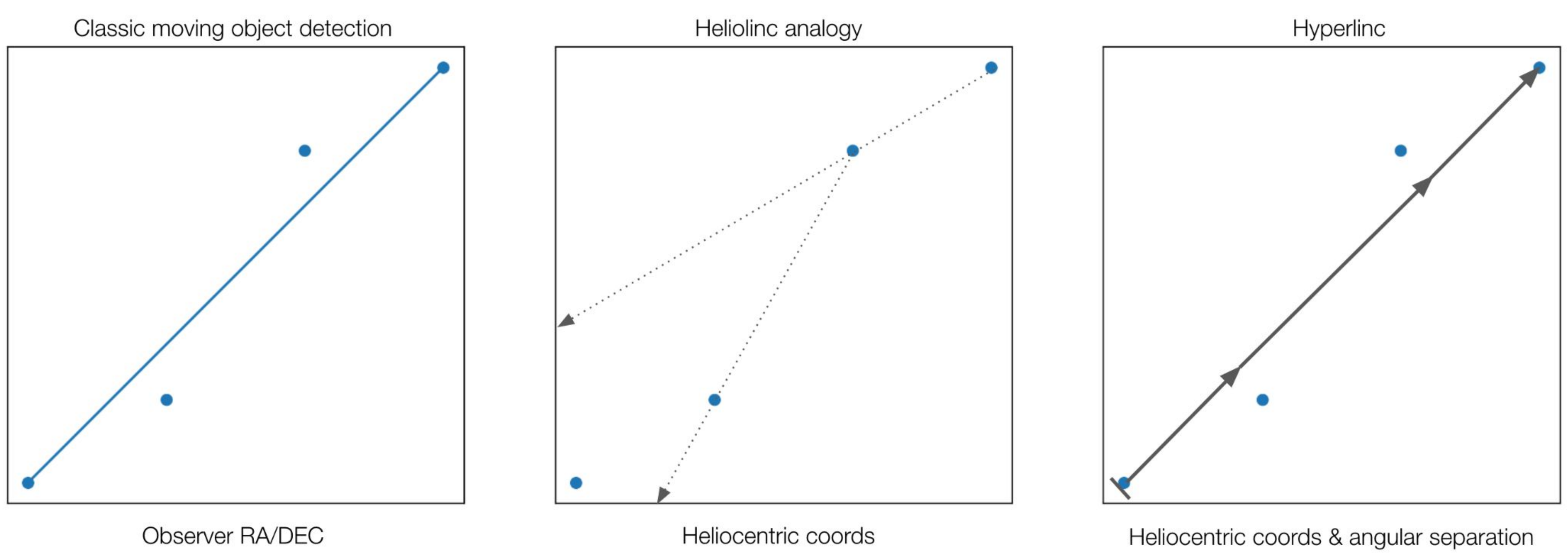
Hyperlinc: Solar System Object Linking via Orbital Plane Clustering

ABSTRACT

Drawing on Heliolinc concepts, this technique starts with heliocentric cartesian projection of transient sources then searches for common relative heliocentric angular motion among sources sharing the same orbital plane. The search is repeated for all hyperplanes containing a sufficient number of transients. In contrast to Heliolinc, this technique averages out astrometric error in a best fit sense and requires no state vector estimation or orbit propagation. And unlike the path of an object traced on the observer relative sky plane used by classic moving object detection methods, the orbital plane of a solar system object in heliocentric coordinates is constant. The physical model underlying the algorithm is validated for 14 and 28 days with JPL Horizons ephemeris data. The algorithm is then applied to Catalina Sky Survey single field data and recovers more than 95% of known objects present in the data. Future work will extend the analysis to multiple nights and make tracklets optional.

LINKING TECHNIQUES AND THEIR SOURCES OF ERROR



LEFT: Classic moving object detection techniques search for linear arrangements of transient sources on the observer sky plane on a single night [1]. A set of sources that fit a line well is known as a tracklet. However, over multiple nights the path of a solar system object on the observer sky plane no longer conforms to a linear model.

MIDDLE: By projecting the observations to well hypothesized heliocentric ranges (guesses essentially), we can look for motion consistent with a solar system object moving in heliocentric coordinates. Heliolinc [2,3,4] does this by using n=2 sized tracklets and solving Lambert's problem to determine an orbit connecting two transient sources and then propagating that orbit to a common reference epoch. Different tracklets that propagate to the same state vector at the reference epoch should belong to the same object. However, astrometric error in tracklet sources compound error in the same way extrapolating a line from two mis-measured points does.

RIGHT: Hyperlinc begins with heliocentric projection of transient sources to an asserted range that yields a position vector just like Heliolinc. But instead of estimating the state vector and propagating an orbit, Hyperlinc finds sets of sources sharing the same orbital plane. Sources that are on the same orbital plane that have common relative heliocentric angular motion belong to the same object. Thus Hyperlinc decomposes the linking problem into: 1) a search for sources on the same orbital plane and 2) a search for common relative heliocentric angular motion on that orbital plane. Each of these can be approached independently as residual minimization problems. The search is then repeated for each orbital plane containing enough sources to constitute a link.

PHYSICAL MODEL VALIDATION

Using heliocentric position vectors from JPL Horizons for the first 1,000 numbered solar system objects, angular RMS deviations from the mean orbital plane are calculated. Next, the RMS of heliocentric angular motion deviations from a linear and quadratic model are calculated relative to the t=0 observation. Mean, 95th percentile and maximum errors are shown for each measure for the first thousand numbered solar system objects. The first table shows 14 days of simulated observations. The second table uses 28 days. All measures are in arcseconds. In summary, the RMS deviation of the unit polar vectors from the mean plane unit vector is negligible and a quadratic model of the heliocentric angular motion of an object as a function of time along its orbital plane has a sub arcsecond RMS error for 95% of the objects modeled over durations of 14 days (LEFT) and 28 days (RIGHT) as shown in the tables below.

14 Days	Mean Plane RMS (arcsec)	Linear Separation RMS (arcsec)	Quadratic Separation RMS (arcsec)
Mean	0.01	3.91	0.02
95th Percentile	0.03	12.06	0.08
Maximum	0.19	46.69	0.63

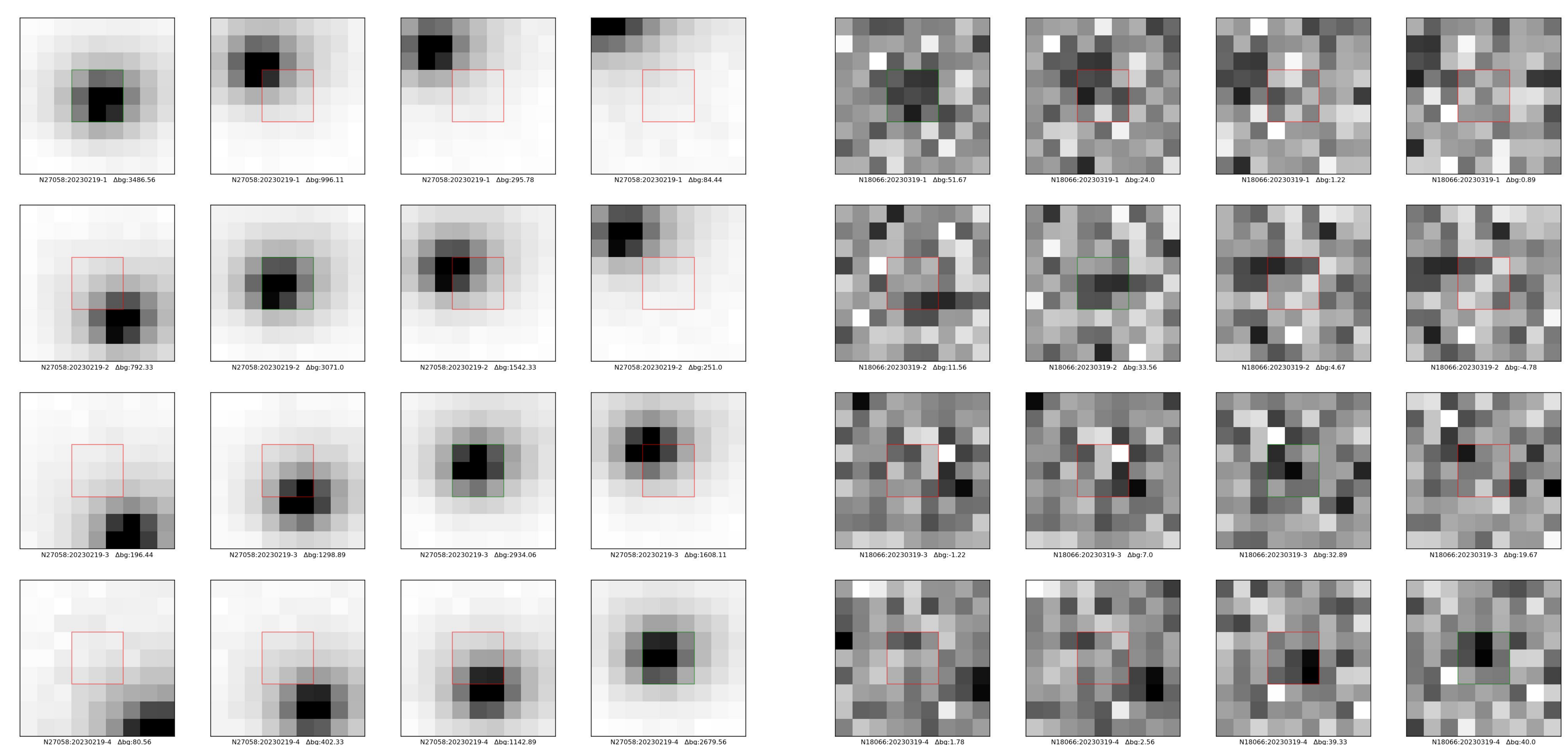
28 Days	Mean Plane RMS (arcsec)	Linear Separation RMS (arcsec)	Quadratic Separation RMS (arcsec)
Mean	0.03	17.11	0.21
95th Percentile	0.10	52.62	0.69
Maximum	0.76	217.24	5.19

RESULTS WITH CATALINA SKY SURVEY DATA

This study looks at the observations from 86 fields CSS [5] observed on March 19th 2023. For the night, Hyperlinc found 6,295 candidate links. 5,871 of those links match known objects and 424 are unknown candidate links. 4,429 out of 4,554 (97.3%) of the known objects with less than 1" astrometric error in the CSS transient source data for the night are recovered by Hyperlinc. 1,442 additional links match known objects with marginal astrometry (between 1"-2" error or very low apparent motion) in one or more of their sources. This may be early evidence of Hyperlinc's ability to average out astrometric error. The table below summarizes the recovery results.

Links Found by Hyperlinc	Links Matching Known Objects with ≤2" Error	Links Matching Known Objects with ≤1" Error	Recoverable Objects with ≤1" Error	Hyperlinc Recovered Known Objects with ≤1" Error	Unknown Links Found by Hyperlinc
6,295	5,871	4,429	4,554	**97.3%**	424

The cutouts below visualize two links in CSS data found by Hyperlinc. Time increases from the top row down. The center square position is constant for each column at the *n*th observation time. Moving objects should be centered on the diagonal and move across the cutouts on a column. The LEFT cutouts below show a bright known object. The RIGHT cutouts show a slow moving low S/N candidate Hyperlinc found that CSS didn't. Almost all unknown links contain spurious sources when visualized, however.



CONCLUSION & FUTURE WORK

Conceptually, Hyperlinc's isolation of transient sources by orbital plane appears distinct enough from existing techniques like Heliolinc, MOPS and THOR [6] to merit further exploration. This study shows that Hyperlinc can achieve greater than 95% recovery of known objects present in the observations with good astrometry for a single night of Catalina Sky Survey data. Hyperlinc is able to recover even more known objects with marginal astrometry beyond that 95%, suggesting the 'best fit' nature of the linking technique has some ability to average out astrometric error. And since the physical model that underlies the algorithm is valid for 14 and 28 days with idealized observations, there is preliminary evidence that it can work for multi-night linking as well. This claim still needs to be tested with real observational data, however.

REFERENCES

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