Creating Dcr-Matched Templates For Image Differencing

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DCR Overview

Refraction deflects the apparent position of sources towards zenith.

The amplitude of refraction depends on environmental factors and the wavelength of incident light.

Differential Chromatic Refraction (DCR) occurs when the index of refraction of the atmosphere changes significantly across the bandwidth of a filter.
DCR Overview

Uncorrected DCR leads to dipoles in difference imaging

Simulated airmass 1.3 observation - zenith template in g-band, with no astrometric calibration.

Calibration and PSF-matching may fix many dipoles, but will make some worse
Uncorrected DCR leads to dipoles in difference imaging.

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Forward Modeling

Each pixel in an image contains flux smeared out along the zenith direction.

A small sub-band of the full filter bandwidth has negligible DCR.

The sub-band model is shifted towards zenith relative to the center of the band.

The original image can be reproduced by shifting and stacking models from all of the sub-bands.
Repeated observations of the same field see the flux smeared in different directions.

Only the direction and magnitude of the shift of the sub-band models depends on the observing conditions.

The pixel values of the models do not change.
Iterative Forward Modeling

Each image is the sum of a series of convolutions with the sub-band models:

$$\sum_{\alpha} B_{i\alpha} \vec{y}_{\alpha} = \vec{s}_i$$

If the convolution kernel $B$ is a shift, then

$$B_{\alpha i}^* B_{i\alpha} = 1$$

Pixel values of each image $i$:

$$\vec{s}_i$$

Pixel values of each sub-band $\alpha$:

$$\vec{y}_\alpha$$

The DCR shift of sub-band $\alpha$ for the observing conditions of image $i$:

$$B_{i\alpha}$$
Iterative Forward Modeling

Each image is the sum of a series of convolutions with the sub-band models:

\[
\sum_{\alpha} B_{i\alpha} \vec{y}_{\alpha} = \vec{s}_i
\]

If the convolution kernel \( B \) is a shift, then

\[
B^*_{\alpha i} B_{i\alpha} = 1
\]

And we can re-write the above equation to solve for a single sub-band model

\[
\vec{y}_{\gamma} = B^*_{\gamma i} \vec{s}_i - B^*_{\gamma i} \sum_{\alpha \neq \gamma} B_{i\alpha} \vec{y}_{\alpha}
\]

=> To solve for \( \vec{y}_{\gamma} \), use an iterative solution and plug in the results from the previous iteration for \( \vec{y}_{\alpha} \)

Note: to prevent oscillating solutions, after each iteration use the average of the new and old solutions for the next iteration
Iterative Forward Modeling

Extension to variable PSFs

A work in progress!

\[ \sum_{\alpha} B_{i\alpha} Q^{(i)} \vec{y}_{\alpha} = P \vec{s}_i \]

- \( P \): PSF of the sub-band models
- \( Q_i \): Measured PSF of each image \( i \)

which gives an iterative solution of

\[ Q^{(i)} \vec{y}_\gamma = B^*_\gamma P \vec{s}_i - B^*_\gamma \sum_{\alpha \neq \gamma} B_{i\alpha} Q^{(i)} \vec{y}_{\alpha} \]

Then, after each iteration we need to solve for \( \vec{y}_{\alpha} \) given solutions of \( Q^{(i)} \vec{y}_{\alpha} \) for each image \( i \).
A simulated g-band airmass 1.3 image.

Stars simulated using Kurucz SEDs and Kolmogorov PSFs

No galaxies, transients, or variable sources
Improved Image Differencing

DCR-matched templates subtract well far from zenith

Fewer 5-σ detections in the difference image than simply using a nearby observation as the template

Simulated observations
Single-filter colors

The DCR sub-band model is built from 8 simulated observations between airmass 2.0 and 1.0.

After detecting sources, forced photometry was run on each of three sub-bands.

The footprint of every detected source is filled with the measured fluxes from the sub-bands, converted to RGB.
Single-filter colors

We know the true spectrum of each simulated source within g-band

So we can compare the measured color between sub-bands to the true color
DCR-matched templates show promise

- By forward-modeling DCR we can use all observations to build our template, regardless of airmass or parallactic angle
  - Can include more effects in the model if known, such as different detector responses
- DCR-matched templates perform well in image differencing
  - We can still use our favorite image differencing algorithm
- We can extract color information from observations taken with a single filter
- Code is in development, but available: https://github.com/lsst-dm/experimental_DCR
Improved Image Differencing

DECam HiTS observation 410998, ‘g’-band with airmass 1.33
The DCR-matched template for 410998, constructed from 12 observations with airmass ranging between 1.13 and 1.77.
Difference image of 410998 with a second DECam observation taken approximately 5 degrees closer to zenith (at airmass 1.23)
Improved Image Differencing

Difference image of 410998 with the DCR-matched template