A MULTI-BAND, MULTI-RESOLUTION DEBLENDER FOR LSST (& OTHERS)

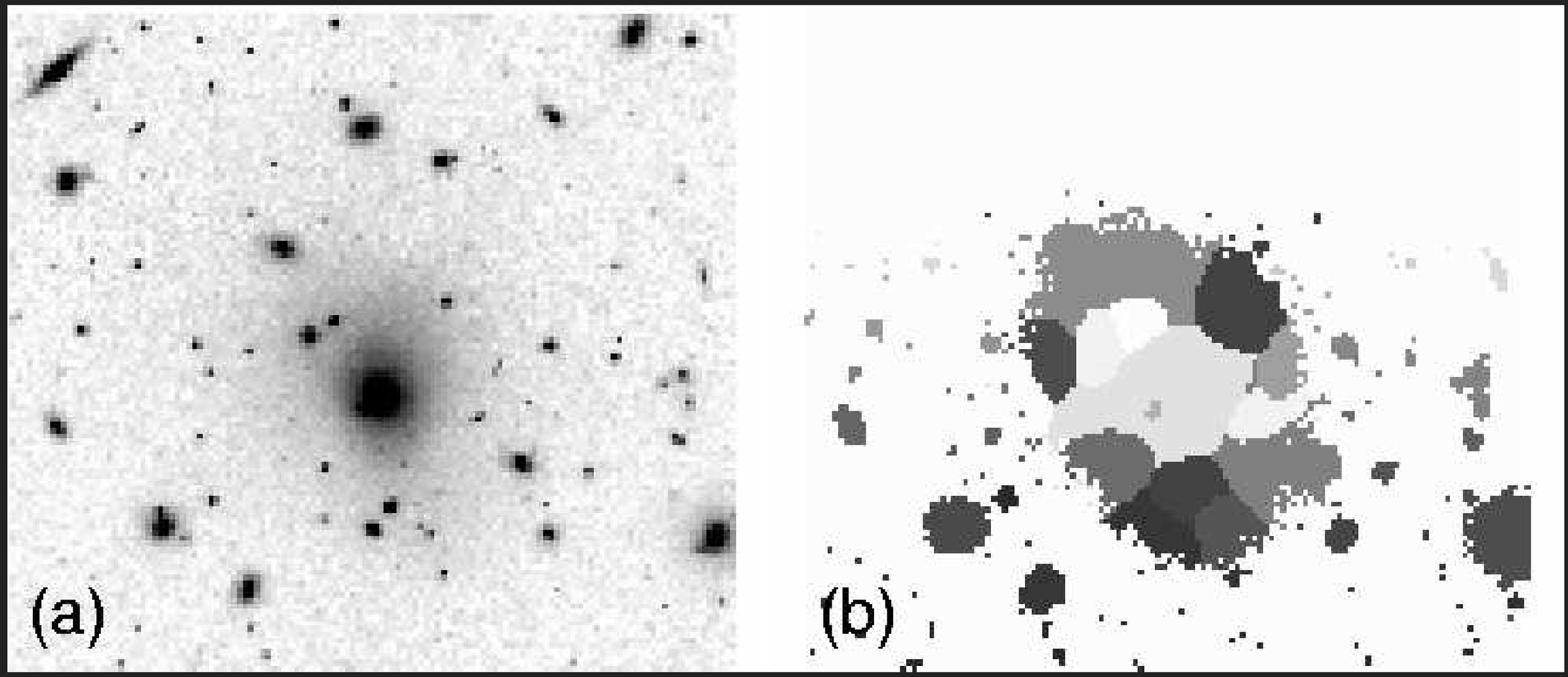
PETER MELCHIOR (PRINCETON)

APPROACHING THE CONFUSION LIMIT

THE CURRENT STATE

- Aperture fluxes do not work!
- Models for blended objects are unstable!
- Heuristic deblenders show flaws!

SEXTRACTOR / SDSS PHOTO



Varela et al. (2009)

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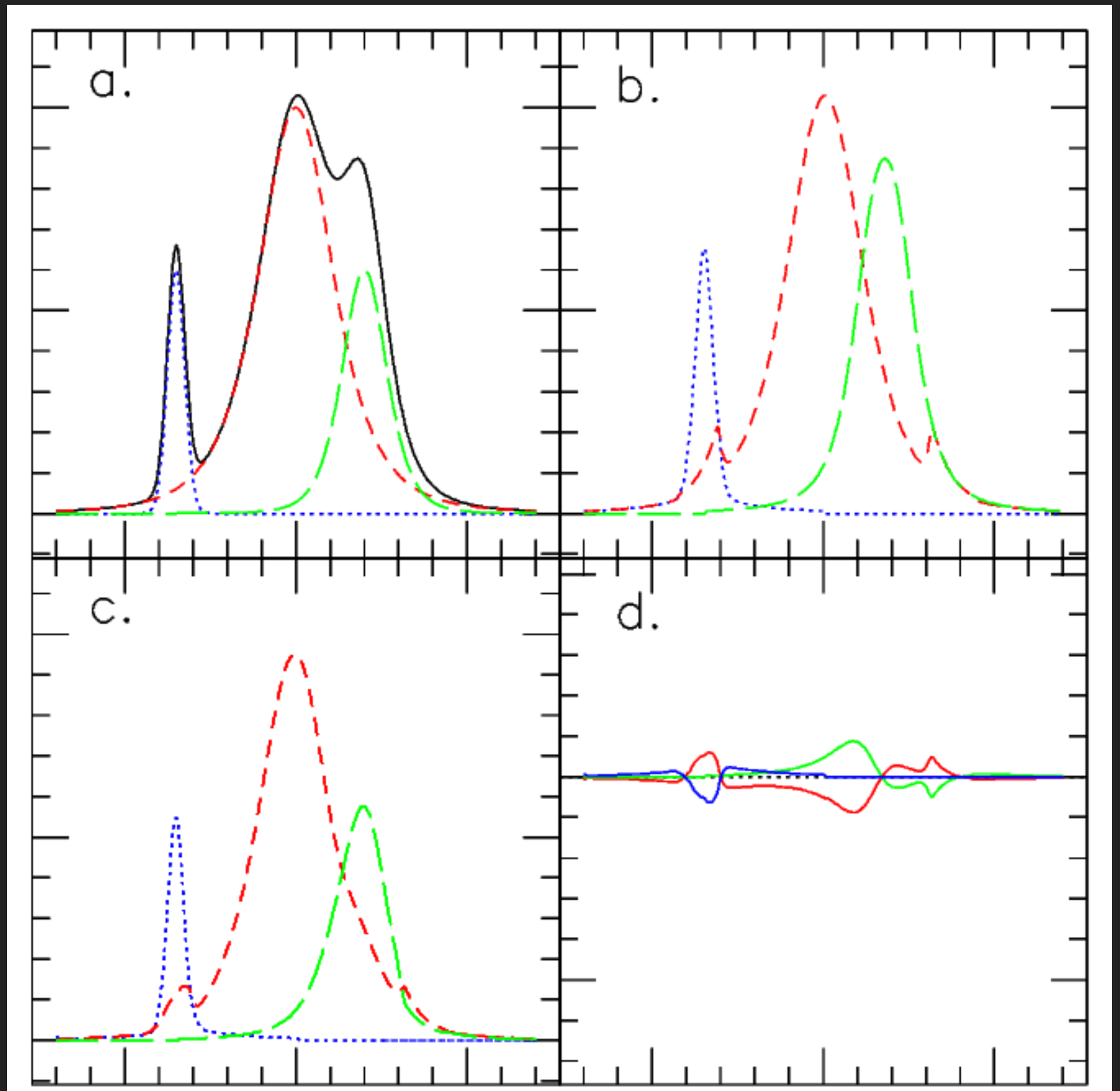
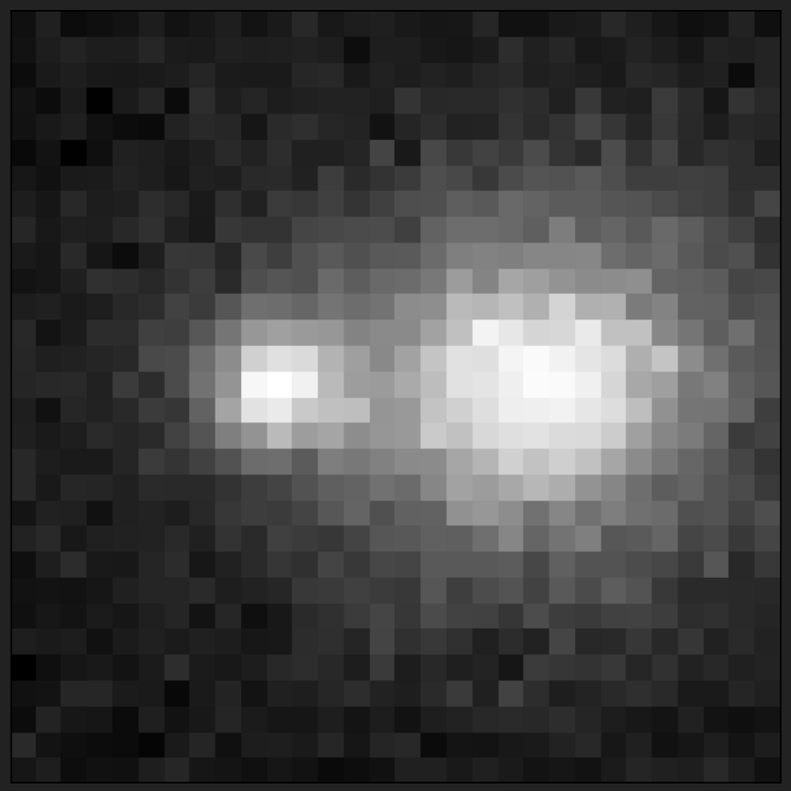


from Robert Lupton

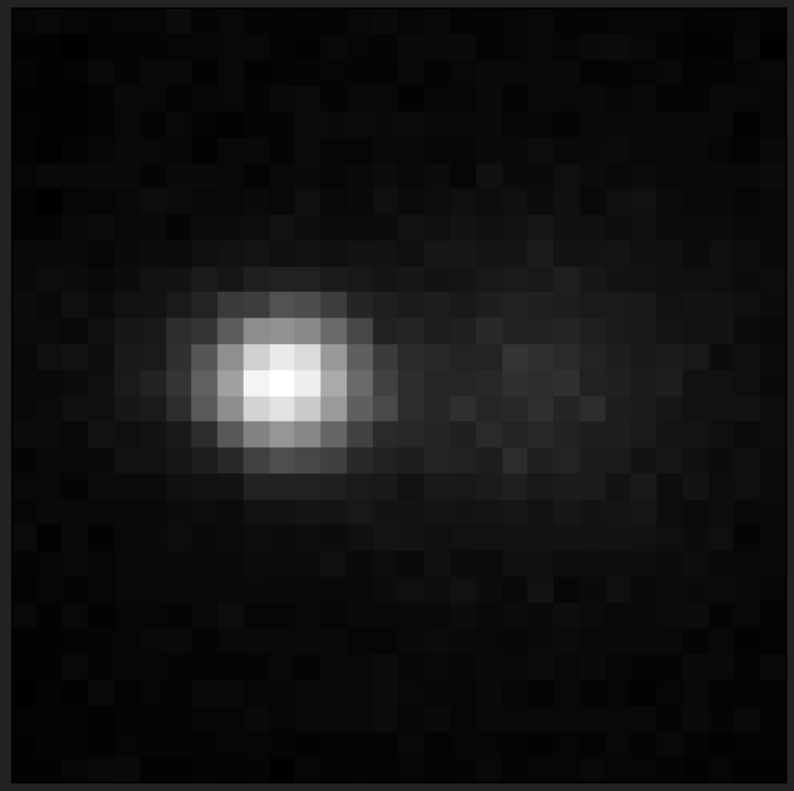
A NEW DEBLENDER

- Star/Galaxy separation is not obvious: non-parametric
- Objects are somehow “compact”, mostly symmetric
- Color should be useful, photo-z are dangerous

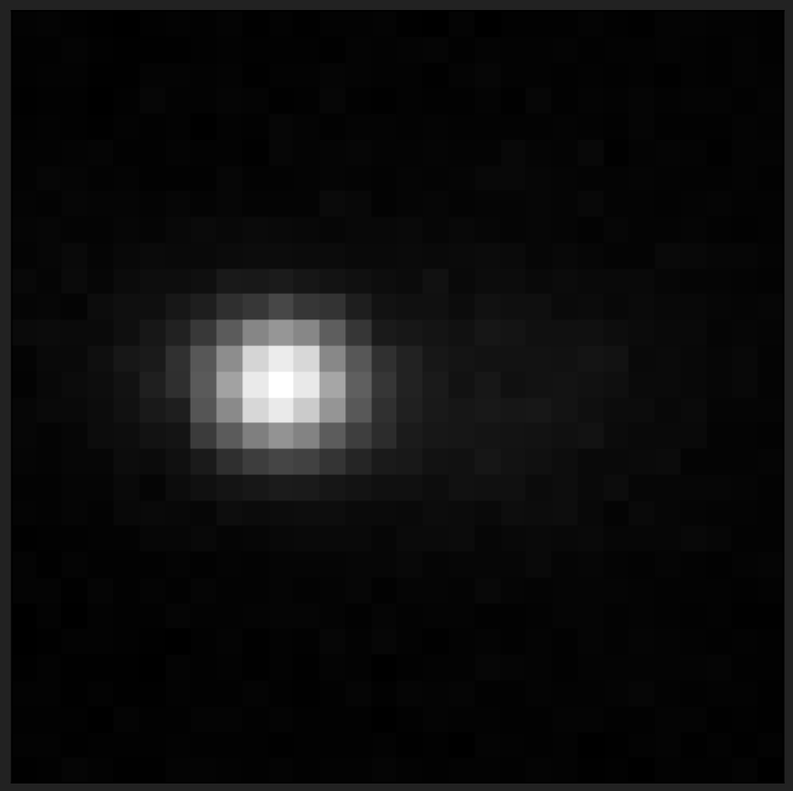
BLIND SOURCE SEPARATION



g band

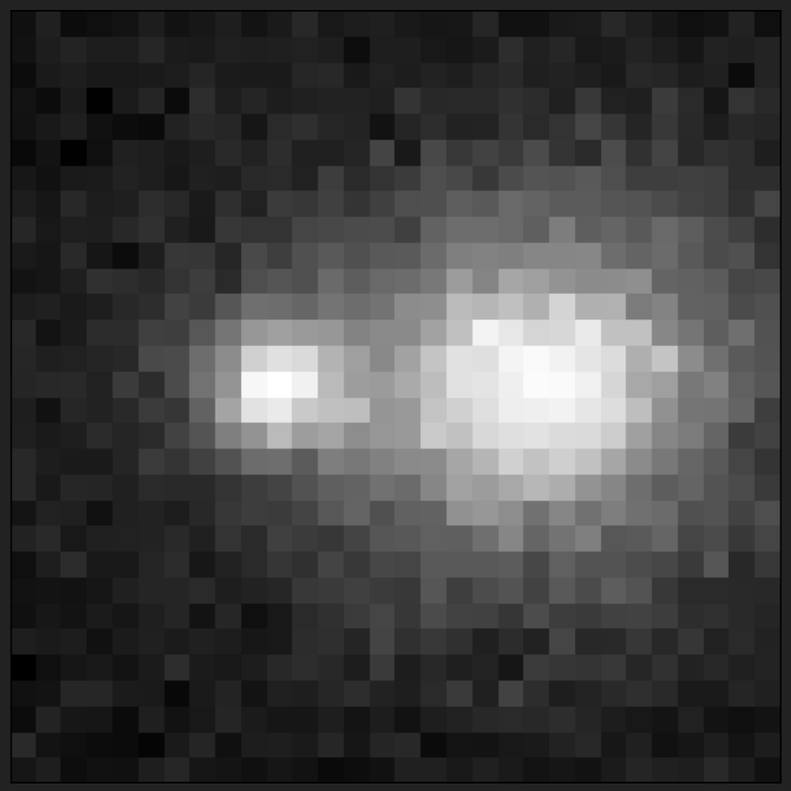


i band



z band

BSS VIA NON-NEGATIVE MATRIX FACTORIZATION



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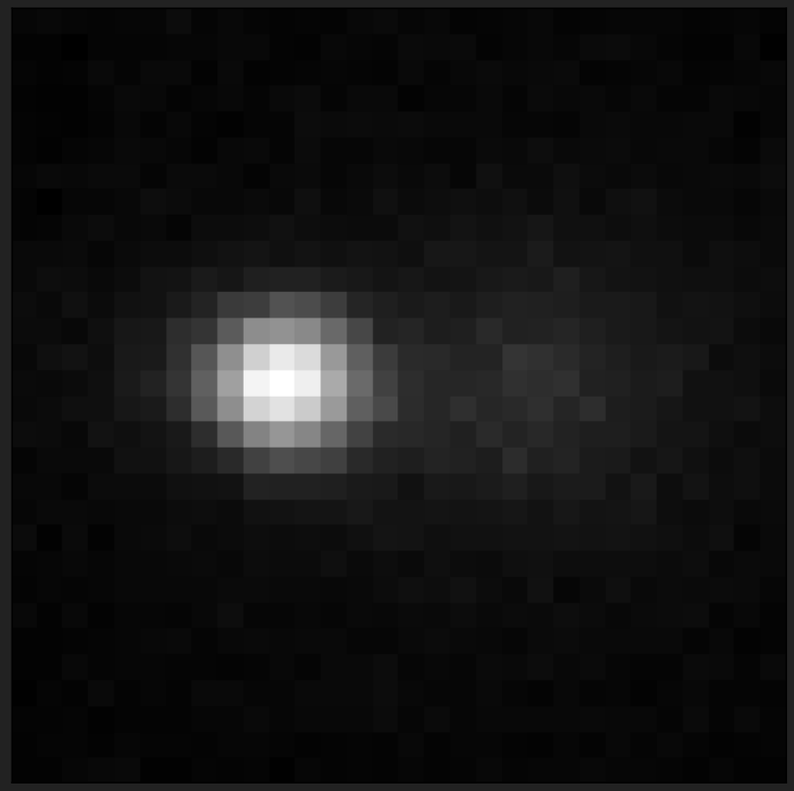

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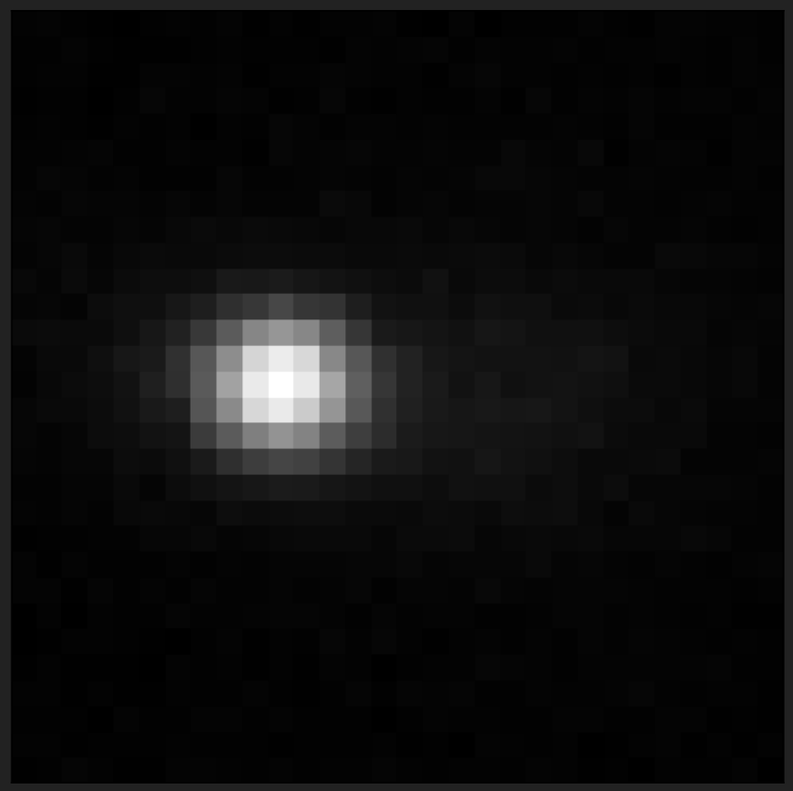

z band

$$\text{scene} = \sum_k \text{SED}_k \times \text{Morphology}_k + \text{noise}$$

$$Y = A \cdot S + \text{noise}$$

$$||Y - A \cdot S||_2^2 + g(A, S)$$

BUT: HOW?

Objective function f(A,S) is quadratic in A and S $||Y - A \cdot S||_2^2$

1. solve for A under constraints (at least non-negative)
2. solve for S under constraints
3. repeat until convergence

Alternating Least-Squares (ALS): does not converge well (if at all)

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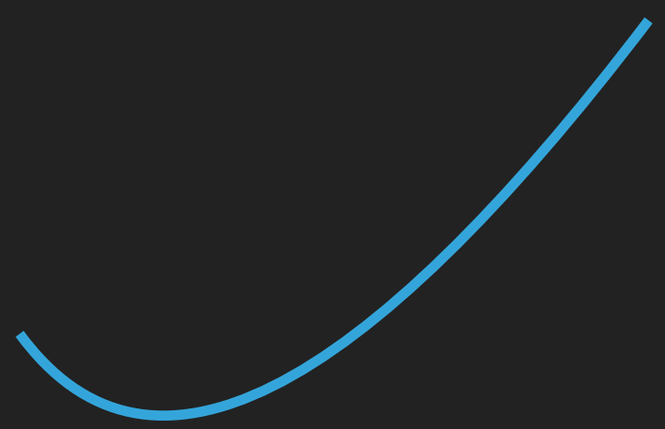

But: f is convex in every argument

EFFICIENT GENERAL SOLVER: ADMM

- Problem: $\min_S \{ f(A, S) + g(\mathsf{G}S) \}$

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- Alternative Direction of Method of Multiplier (ADMM):

$$S^{k+1} := \underset{S}{\operatorname{argmin}} \left\{ f(S \mid A) + \lambda^k \mathsf{G}S + \frac{1}{2\rho} ||\mathsf{G}S - Z^k||_2^2 \right\}$$

$$Z^{k+1} := \underset{Z}{\operatorname{argmin}} \left\{ g(Z) + \lambda^{kT} Z + \frac{1}{2\rho} ||\mathsf{G}S^{k+1} - Z||_2^2 \right\}$$

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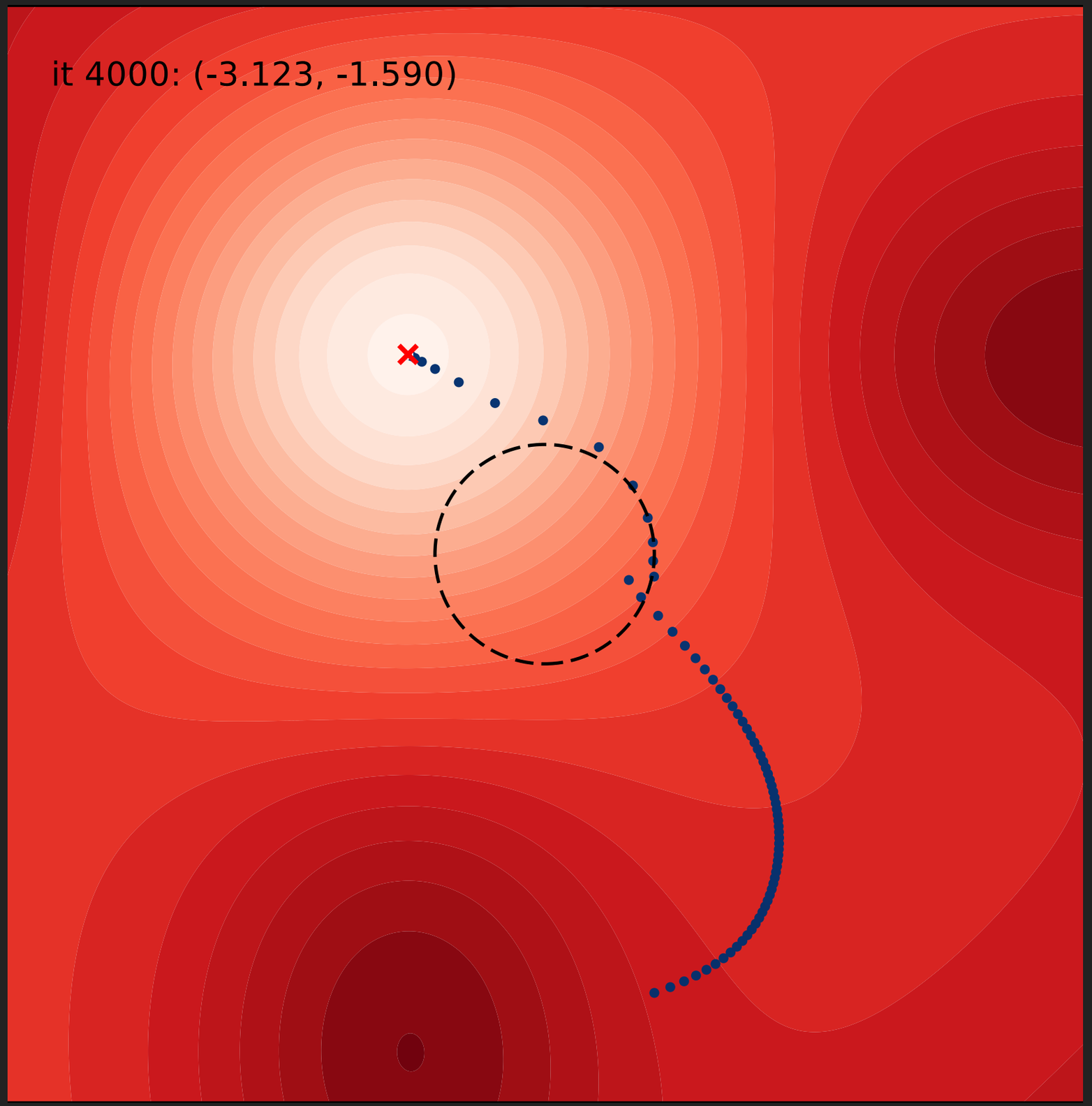
- Often per-iteration updates are simple & analytic:

$$S^{k+1} := \nabla_S \left(f(A,S) + \frac{\mu}{\rho} \mathsf{G}^T \left(\mathsf{G}S^k - Z^k + \lambda^k \right) \right)$$

EFFICIENT GENERAL SOLVER: ADMM



THE INTERFACE IS SIMPLE!

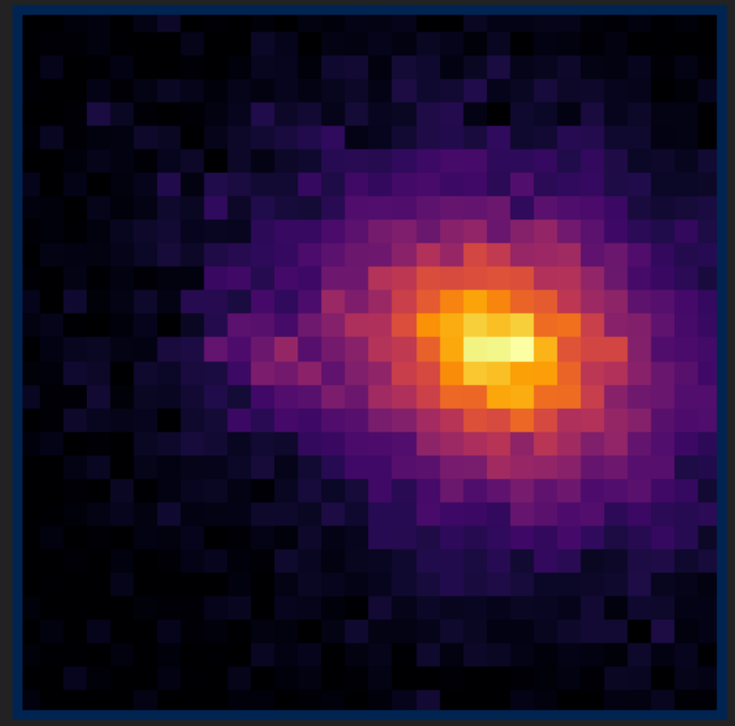
```
1   def grad_f(xy):
2       """Gradient of f"""
3       x,y = xy
4       return np.array([2*x, 2*y)])
5
6   def prox_circle(xy, step):
7       """Projection onto circle"""
8       center = np.array([0,0])
9       dxy = xy - center
10      radius = 0.5
11      # exclude interior of circle
12      if (dxy**2).sum() < radius**2:
13          phi = np.arctan2(dxy[1], dxy[0])
14          return center + radius*np.array([np.cos(phi), np.sin(phi)])
15      else:
16          return xy
```

DEBLENDING GALAXIES WITH NMF MELCHIOR ET AL. (IN PREP)

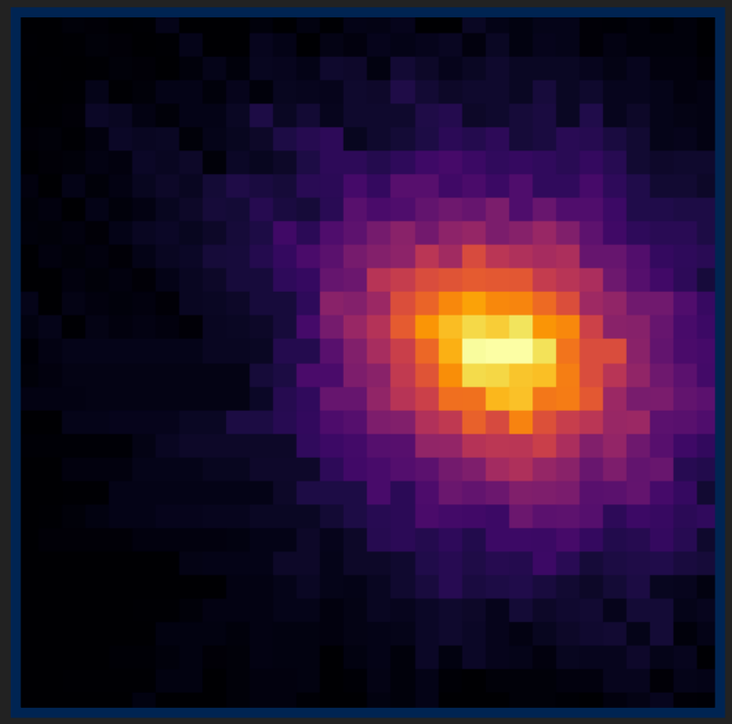
$$g(A, S) \rightarrow \sum_k g_k(A_k) + h_k(\mathsf{L}S_k)$$

- per-object constraints with linear operators: gradients, symmetry, FFT …
- SED: sum=1, particular colors, distribution of observed colors

3-band RGB



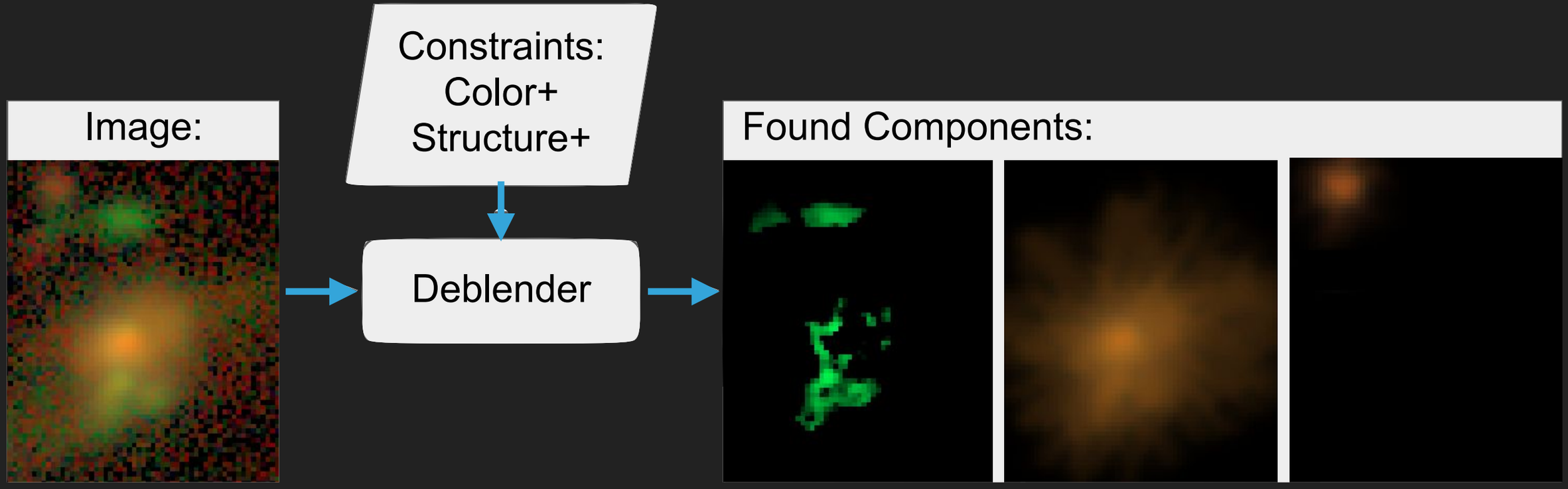
NMF: no constraint



NMF: with monotonicity

AGN JET VS HOST GALAXY

- 5-band HSC data
- Model: Jet + 2-component host + neighbor
- PSF matching on the fly



by Max Jerdee (Princeton)

GETTING REAL

- Sparsity vs PSF convolution // on-the-fly PSF matching

$$||Y - A \cdot P \cdot S||_2^2$$

- Shift operators for centroiding // moving objects

$$||Y - A \cdot T \cdot P \cdot S||_2^2$$

- Constraints operators are identical, only likelihood term get adjusted

THE CASE FOR GROUND & SPACE

DES data from Melchior et al. (2015)

THE CASE FOR GROUND & SPACE

CLASH WFC3/IR data

CONCLUSIONS

- Whenever you have an additive mixture situation: think NMF
- Soft priors and hard constraints can be implemented
- NMF code will be released with algorithm paper in next few weeks
- Deblender: multi-band by design, multi-resolution-ready
- Standalone version & integration into the LSST stack
- Post-doc position available

NMF FOR HYPER-SPECTRAL UNMIXING

210 wavelengths
500nm to 2 micron

NMF FOR HYPER-SPECTRAL UNMIXING

Shadow

Road

Trees

Concrete

Roof

Grass

Path

Water

Moolekamp &
Melchior (in prep.)